Magnetic Field Formulation (variable )

## Dimensional induction equation for variable properties

Maxwell's equations (while neglecting the displacement current):

Solving for the electric field in terms of the current in Ohm's law yields

Plugging this into Faraday's Law yields

Distributing we have

Applying Ampere's Law to the current yields

This is the final form. Alternatively, if we would like to write the curls in terms of dot products and gradients, we can proceed as follows. Using the vector identity

On the term, we have:

Now, looking at the last term on the RHS before using Ampere's Law. The product of a scalar and vector field for the curl operator is:

Therefore, since , we have

Substituting in the current from Ampere's Law we have

Again, using

to the current on the LHS we have

Using the vector identity

On the first term, we have:

Now, using

Using the identity

We have

Expanding the left term we have

We may interpret this as

Note: The instructions from the vector form, , of the term is not explicitly clear. From here on, it will be assumed that these two expressions are synonymous.

Therefore we have

Therefore, our final form is:

Distributing some terms we have

Distributing the negative sign

Since and assuming , yields the

Where the terms in red are the source terms associated with variation in with space.

## Dimensionless induction equation for variable properties inside a fluid

Introducing the following reference values to non-dimensionalize by

Removing the red color, and substituting our dimensionless numbers in, we have

Making the LHS of order one yields

Introducing the characteristic magnetic Reynolds number

Yields

Removing the asterisks, yields

This is the form of the dimensionless induction equation for variable properties.

Where, again, the terms in red are the source terms associated with variation in with space.

In addition, the current can be non-dimensionalized as follows

Therefore we have

So the non-dimensional current is

## Dimensionless induction equation for variable properties inside a fluid in terms of current

Introducing the following reference values to non-dimensionalize by

Making the LHS of order one yields

Removing the asterisks, yields

Where

This is the form of the dimensionless induction equation for variable properties.